Instability of rotating black hole in a limited form of $f(R)$ gravity

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Abstract

We investigate the stability of $f(R)$-rotating (Kerr) black hole obtained from a limited form of $f(R)$ gravity. In order to avoid the difficulty of handling fourth order linearized equations, we transform this form of $f(R)$ gravity into the scalar-tensor theory by introducing two auxiliary scalars. In this case, the linearized curvature scalar equation leads to a massive scalaron equation. It turns out that the $f(R)$-rotating black hole is unstable because of the superradiant instability known as the black-hole bomb idea.

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1 Introduction

Modified gravity theories, $f(R)$ gravities [1, 2, 3] have much attentions as one of strong candidates for explaining the current accelerating universe [4]. $f(R)$ gravities can be considered as Einstein gravity with an additional scalar field. For example, the metric-$f(R)$ gravity is equivalent to the $\omega_{\rm BD} = 0$ Brans-Dicke (BD) theory with the potential [5]. It was shown that the equivalence principle test allows $f(R)$ gravity models that are nearly indistinguishable from the ΛCDM model in the background universe evolution [6]. However, this does not imply that there is no difference in the dynamics of perturbations [7]. It is worth noting that the perturbation distinguishes between Einstein and $f(R)$ gravities.

In order for $f(R)$ gravities to be acceptable, they obey certain minimal requirements for theoretical viability [2, 5]. Three important requirements are included: (i) they possess the correct cosmological dynamics, (ii) they are free from instabilities, tachyon and ghosts [8, 6], (iii) they attain the correct Newtonian and post-Newtonian limits.

On the other hand, the Schwarzschild-de Sitter black hole was obtained for a constant curvature scalar from $f(R)$ gravity [7]. A black hole solution was obtained from $f(R)$ gravities by requiring the negative constant curvature scalar $R = R_0$ [9]. For $1+f'(R_0) > 0$, this black hole is similar to the Schwarzschild-AdS (SAdS) black hole. In order to obtain the (constant curvature) black hole solution from $f(R)$ gravity coupled to the matters of the Maxwell [9] and Yang-Mills fields [10], the trace of its stress-energy tensor $T_{\mu\nu}$ should be zero. Interestingly, it was pointed out that the Kerr solution could be obtained from a limited form of $f(R)$ gravity [11]. Later on, it was argued that perturbed Kerr black hole can discriminate Einstein and $f(R)$ gravities [12].

All astrophysical black holes belong to the rotating black hole. A black hole solution should be stable against the external perturbations because it stands as a physically realistic object [13]. Studies of stability of Kerr black hole are not as straightforward, because it is axially symmetric black hole and thus, the decoupling process seems to be complicated. The Kerr black hole has been proven to be stable against gravitational fields [14, 15, 16] and massless scalar [17]. However, there exist unstable modes when considering a massive scalar due to the superradiance [18]. Furthermore, it seems that the stability analysis of $f(R)$-rotating black hole is a formidable task because $f(R)$ gravity contains fourth order derivatives in the linearized equations [12].

In this work, we investigate the stability of $f(R)$-rotating (Kerr) black hole arisen from

a limited form (2.6) of $f(R)$ gravity [11]. This work will be interesting because if $f(R)$ gravity is considered as a viable theory, it is responsible to explain the present accelerating universe as well as the formation of rotating black holes. We transform the form of $f(R)$ gravity into the scalar-tensor theory to avoid fourth order derivative terms by introducing two auxiliary scalars. Then, the linearized curvature scalar equation becomes a massive scalaron equation, indicating that all linearized equations are second order. Using the stability analysis of Kerr black hole in the massive Klein-Gordon equation, we show clearly that the $f(R)$-rotating black hole is unstable against the scalaron perturbation.

2 Perturbation of $f(R)$ black holes

We start with $f(R)$ gravity without any matter fields whose action is given by

$$S_f = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R), \tag{2.1}$$

where $\kappa^2 = 8\pi G$. The Einstein equation takes the form

$$R_{\mu\nu}f'(R) - \frac{1}{2}g_{\mu\nu}f(R) + \Big(g_{\mu\nu}\nabla^2 - \nabla_\mu\nabla_\nu\Big)f'(R) = 0, \tag{2.2}$$

where $'$ denotes the differentiation with respect to its argument. It is well-known that (2.2) has a solution with constant curvature scalar $R = \bar{R}$. In this case, (2.2) can be written as

$$\bar{R}_{\mu\nu}f'(\bar{R}) - \frac{1}{2}g_{\mu\nu}f(\bar{R}) = 0, \tag{2.3}$$

and thus, the trace of (2.3) becomes

$$\bar{R} = \frac{2f(\bar{R})}{f'(\bar{R})} \equiv 4\Lambda_f \tag{2.4}$$

with Λ_f the cosmological constant due to the $f(R)$ gravity. Substituting this expression into (2.3), one obtains the Ricci tensor

$$\bar{R}_{\mu\nu} = \frac{f(\bar{R})}{2f'(\bar{R})}\bar{g}_{\mu\nu} = \Lambda_f\bar{g}_{\mu\nu}. \tag{2.5}$$

In order to find the Kerr black hole solution, we have to choose a non-pathologically functional form of $f(R)$ as [11]

$$f(R) = a_1R + a_2R^2 + a_3R^3 + \cdots \tag{2.6}$$

which is surely a Talyor series around $R = \bar{R} = 0$. We should mention that this form of $f(R)$ gravity is not general, but a small subset of $f(R)$ gravities. We check that $f(0) = 0,\ f'(0) = a_1,\ f''(0) = 2a_2$ at $R = \bar{R} = 0$, which provides either the Schwarzschild or Kerr black hole solution when choosing $\Lambda_f = 0$.

In this work, we use the Boyer-Lindquist coordinates to represent an axis-symmetric Kerr black hole solution with mass M and angular momentum J [19],

$$
\begin{aligned}
ds^2_{\rm Kerr} &= -\left(1-\frac{2Mr}{\rho^2}\right)dt^2 - \frac{2Mra\sin^2\theta}{\rho^2}\,2dtd\phi + \frac{\rho^2}{\Delta}\,dr^2 \\
&\quad +\rho^2 d\theta^2 + \left(r^2+a^2+\frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta\, d\phi^2
\end{aligned} \tag{2.7}
$$

with

$$
\Delta = r^2 + a^2 - 2Mr\,, \qquad \rho^2 = r^2 + a^2\cos^2\theta\,, \qquad a = \frac{J}{M}. \tag{2.8}
$$

In the limit of $a \to 0$, (2.7) recovers the Schwarzschild black hole, while $a \to 1$ goes to the extremal Kerr black hole. The zeros of Δ, two horizons are located at

$$
r_\pm = M \pm \sqrt{M^2 - a^2} \tag{2.9}
$$

and the angular velocity at the event horizon takes the form

$$
\Omega = \frac{a}{2Mr_+}. \tag{2.10}
$$

Now we introduce the perturbation around the Kerr black hole to study stability of the black hole

$$
g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \tag{2.11}
$$

Hereafter we denote the background quantities with the "overbar". Then, the linearized equation around $f(R)$-rotating black hole becomes

$$
\begin{aligned}
&\bar{\nabla}^\rho\bar{\nabla}_\mu h_{\nu\rho} + \bar{\nabla}^\rho\bar{\nabla}_\nu h_{\mu\rho} - \bar{\nabla}^2 h_{\mu\nu} - \bar{\nabla}_\mu\bar{\nabla}_\nu h - \bar{g}_{\mu\nu}\left(\bar{\nabla}^\alpha\bar{\nabla}^\beta h_{\alpha\beta} - \bar{\nabla}^2 h\right) \\
&\qquad + \left[\frac{2}{3m_f^2}\right]\left(\bar{g}_{\mu\nu}\bar{\nabla}^2 - \bar{\nabla}_\mu\bar{\nabla}_\nu\right)\left(\bar{\nabla}^\alpha\bar{\nabla}^\beta h_{\alpha\beta} - \bar{\nabla}^2 h\right) = 0
\end{aligned} \tag{2.12}
$$

with the mass squared m_f^2 defined by

$$m_f^2 = \frac{f'(0)}{3f''(0)}. \tag{2.13}$$

Taking the trace of (2.12) with $\bar{g}^{\mu\nu}$, one has the fourth order equation for $h_{\mu\nu}$

$$\Big(\bar{\nabla}^2 - m_f^2\Big)\delta R(h) = 0 \to \Big(\bar{\nabla}^2 - m_f^2\Big)\Big(\bar{\nabla}^\alpha\bar{\nabla}^\beta h_{\alpha\beta} - \bar{\nabla}^2 h\Big) = 0. \tag{2.14}$$

At this stage, we note that it is not easy to make a further progress on the perturbation analysis because there exist fourth order derivatives. We mention that for the Einstein gravity with $f(R) = R$, $f'(0) = 1$ and $f''(0) = 0$. In this case, one finds the equation for linearized curvature scalar: $\delta R(h) = 0$, which means that $\delta R(h)$ is not a physically propagating mode. Actually, this equation leads to one constraint as

$$\bar{\nabla}^\alpha\bar{\nabla}^\beta h_{\alpha\beta} = \bar{\nabla}^2 h \tag{2.15}$$

which will also be recovered from the transverse gauge.

Up to now, we did not fix any gauge. We would like to comment on the linearized equation by choosing the Lorentz gauge [20]

$$\bar{\nabla}_\nu h^{\mu\nu} = \frac{1}{2}\bar{\nabla}^\mu h. \tag{2.16}$$

Under this gauge-fixing, the linearized equation (2.12) takes a simple form

$$\bar{\nabla}^2\tilde{h}_{\mu\nu} + 2\bar{R}_{\mu\rho\nu\sigma}\tilde{h}^{\rho\sigma} + \frac{1}{3m_f^2}\Big(\bar{g}_{\mu\nu}\bar{\nabla}^2 - \bar{\nabla}_\mu\bar{\nabla}_\nu\Big)\bar{\nabla}^2\tilde{h} = 0 \tag{2.17}$$

with the trace-reversed perturbation $\tilde{h}_{\mu\nu} = h_{\mu\nu} - h\bar{g}_{\mu\nu}/2$ and $1/3m_f^2 = \lambda$ [12]. This equation was importantly used to mention that perturbed Kerr black holes obtained from $f(R)$ gravity can probe deviations from the Einstein gravity [12]. Even though equation (2.17) is simpler than (2.12), it is a non-trivial task to decouple odd and even perturbations around the Kerr black hole, arriving at two fourth order equations hopefully. Crucially, we do not know how to solve the fourth order equation (2.17) around the $f(R)$-rotating black hole. On the other hand, we may choose the transverse gauge which works well for studying the graviton propagations on the the Minkowski and AdS$_4$ spacetime backgrounds [21, 22]

$$\bar{\nabla}_\mu h^{\mu\nu} = \bar{\nabla}^\nu h, \tag{2.18}$$

which leads to (2.15) when operating $\bar{\nabla}$ on both sides. Using the relation (2.15), one immediately finds that the effect of $f(R)$ gravity [$2/3m_f^2$-term in (2.12)] disappears because of $\delta R(h) = 0$, leading to the Einstein gravity. Hence, the gauge-fixing for stability analysis of the $f(R)$-rotating black hole differs from that for the graviton propagations on the AdS, dS, and Minkowski spacetimes [20]. We note that choosing an appropriate gauge-fixing cannot resolves the difficulty with $f(R)$ gravities. It seems that the best way to resolve the difficulty confronting with the fourth order equation is to translate the fourth order equation into the second order equations by introducing auxiliary scalar fields. In other words, we must make a transformation from the limited form of $f(R)$ gravity to the scalar-tensor theory (like Brans-Dicke theory) to perform the stability analysis of $f(R)$-rotating black hole.

3 Perturbation of the scalar-tensor theory

In this section, we will develop the perturbation analysis around the $f(R)$-rotating black holes (2.7) in the different frame, the scalar-tensor theory. Introducing two auxiliary fields ϕ and A, one can rewrite the action (2.1) as [23, 24]

$$S_{st} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \Big[\phi \left(R - A \right) + f_A(A) \Big]. \tag{3.1}$$

Varying for the fields ϕ and A leads to two equations

$$R = A, \quad \phi = f'_A(A). \tag{3.2}$$

Note that using (3.2), the action (3.1) recovers the original action (2.1). On the other hand, the equation of motion for the metric tensor can be obtained by

$$f'_A(A) R_{\mu\nu} - \frac{f_A(A)}{2} g_{\mu\nu} + \Big(g_{\mu\nu} \nabla^2 - \nabla_\mu \nabla_\nu \Big) f'_A(A) = 0. \tag{3.3}$$

Considering a constant curvature scalar $R = \bar{R} = \bar{A}$ together with $\bar{\phi} = f'_A(\bar{A}) =$ const, Eq.(3.3) becomes

$$f'_A(\bar{A}) \bar{R}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} f_A(\bar{A}) = 0. \tag{3.4}$$

Taking the trace of (3.4) leads to

$$\bar{R} = \frac{2 f_A(\bar{A})}{f'_A(\bar{A})} \equiv 4\Lambda_A. \tag{3.5}$$

Substituting this expression into (3.4), one finds the Ricci tensor which determines the maximally symmetric Einstein spaces including Minkowski space

$$\bar{R}_{\mu\nu} = \frac{1}{2}\frac{f_A(\bar{A})}{f'_A(\bar{A})}\bar{g}_{\mu\nu} = \Lambda_A \bar{g}_{\mu\nu}. \tag{3.6}$$

At this stage, we introduce a specific form of $f_A(A)$ inspired from (2.6) [11]

$$f_A(A) = a_1 A + a_2 A^2 + a_3 A^3 \cdots, \tag{3.7}$$

where $a_1, a_2, a_3, \cdots$ are arbitrary constants. Their mass dimensions are $[a_1] = 0,\ [a_2] = -2$ because $[f_A(A)] = 2,\ [A] = 2$, and $[\phi] = 2$. In this work, we confine ourselves to the asymptotically flat spacetimes with $\Lambda_A = 0$ which accommodates the $f(R)$-rotating black hole (2.7). In this case, we have to choose $\bar{A} = 0$ and thus,

$$f_A(0) = 0, \quad f'_A(0) = a_1 > 0,\ f''_A(0) = 2a_2 > 0. \tag{3.8}$$

Now we are in a position to study the perturbation around the Kerr black hole (2.7). In addition to (2.11), from (3.2), we have

$$\bar{R} + \delta R(h) = \bar{A} + \delta A, \quad \bar{\phi} + \delta\phi = f'_A(\bar{A}) + f''_A(\bar{A})\delta A, \tag{3.9}$$

which leads to

$$\delta R(h) \to \delta A, \quad \delta\phi \to f''_A(\bar{A})\delta A. \tag{3.10}$$

Thus, instead of $\delta R(h)$-$\delta\phi$, we use δA as a perturbed field in addition to $h_{\mu\nu}$. We mention that if one uses $\delta R(h)$-$\delta\phi$, one gets the same linearized equations. For this, see the appendix of Ref.[25].

Using (3.10), the linearized equations to (3.3) reduces to

$$\delta R_{\mu\nu}(h) = \left[\frac{1}{3m_A^2}\right]\bar{\nabla}_\mu\bar{\nabla}_\nu\delta A + \frac{1}{6}\bar{g}_{\mu\nu}\delta A, \tag{3.11}$$

where the scalaron mass squared is given by

$$m_A^2 = \frac{f'_A(0)}{3f''_A(0)}. \tag{3.12}$$

Taking the trace of (3.11) and using (3.10) leads to the massive scalaron equation

$$\left(\bar{\nabla}^2 - m_A^2\right)\delta A = 0. \tag{3.13}$$

Since the mass dimension of the linearized scalaron is two ($[\delta A] = 2$), it would be better to write the canonically linearized equations by introducing a dimensionless scalaron $\delta\tilde{A}$ defined by

$$\delta\tilde{A} = \frac{\delta A}{3m_A^2}. \tag{3.14}$$

Finally, we arrive at two linearized equations

$$\left(\bar{\nabla}^2 - m_A^2\right)\delta\tilde{A} = 0, \tag{3.15}$$

$$\delta R_{\mu\nu}(h) = \bar{\nabla}_\mu\bar{\nabla}_\nu\delta\tilde{A} + \left[\frac{m_A^2}{2}\right]\bar{g}_{\mu\nu}\delta\tilde{A}, \tag{3.16}$$

which are our main equations to carry out the stability analysis of $f(R)$-rotating black hole. It is a nontrivial task to perform the stability analysis of $f(R)$-rotating black hole based on the metric-perturbing equation (3.16) since the decoupling process seems to be complicated. However, we expect that the right hand side of scalaron coupling terms in (3.16) will not change the stability of $f(R)$-rotating black hole [25, 26]. In this work, we focus on the scalaron equation (3.15) because it shows the feature of $f(R)$ gravities.

4 Stability analysis of $f(R)$-rotating black hole

Considering the axis-symmetric background (2.7), it is convenient to separate the scalaron into [27]

$$\delta\tilde{A}(t,r,\theta,\phi) = e^{-i\omega t + im\phi}S_l^m(\theta)R(r)\,, \tag{4.1}$$

where $S_l^m(\theta)$ are spheroidal angular functions, and the azimuthal number m takes on (positive or negative) integer values. Also, it is enough to consider positive ω's in (4.1). Plugging this into the scalaron (Klein-Gordon) equation (3.15), we get the angular and radial wave equations for $S_l^m(\theta)$ and $R(r)$ as

$$\begin{aligned} &\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta S_l^m\right) \\ &\qquad + \left[a^2(\omega^2 - m_A^2)\cos^2\theta - \frac{m^2}{\sin^2\theta} + A_{lm}\right]S_l^m = 0\,, \end{aligned} \tag{4.2}$$

$$\begin{aligned} &\Delta\partial_r\left(\Delta\partial_r R\right) + \big[\omega^2(r^2+a^2)^2 - 4Mam\omega r + a^2m^2 \\ &\qquad\qquad -\Delta(a^2\omega^2 + m_A^2 r^2 + A_{lm})\big]R = 0\,, \end{aligned} \tag{4.3}$$

where A_{lm} is the separation constant that allows the split of the wave equation [28, 29]

$$A_{lm} = l(l+1) + \sum_{k=1}^{\infty} c_k a^{2k} (m_A^2 - \omega^2)^k \,. \tag{4.4}$$

The radial Teukolsky equation takes the Schrödinger form

$$-\frac{d^2\tilde{R}}{dr^{*2}} + V(r,\omega)\tilde{R} = \omega^2\tilde{R}, \quad \tilde{R} = \sqrt{r^2+a^2}R, \tag{4.5}$$

where the tortoise r_* coordinate is defined by $dr^* = \frac{r^2+a^2}{\Delta}dr$ and the ω-dependent potential $V(r,\omega)$ is given by

$$\begin{aligned} V(r,\omega) &= \frac{\Delta m_A^2}{r^2+a^2} + \frac{4Mram\omega - a^2m^2 + \Delta[A_{lm} + (\omega^2 - m_A^2)a^2]}{(r^2+a^2)^2} \\ &+ \frac{\Delta(3r^2 - 4Mr + a^2)}{(r^2+a^2)^3} - \frac{3\Delta^2 r^2}{(r^2+a^2)^4}. \end{aligned} \tag{4.6}$$

Its asymptotic forms are given by

$$V \to \omega^2 - m_A^2, \quad r^* \to \infty (r \to \infty), \tag{4.7}$$

$$V \to (\omega - m\Omega)^2, \quad r^* \to -\infty (r \to r_+). \tag{4.8}$$

Near the horizon at $r = r_+$ and spatial infinity at $r = \infty$, the scalaron takes the form

$$\tilde{R} = \mathcal{T} e^{-i(\omega - m\Omega)r^*} \ , \ r^* \to -\infty \tag{4.9}$$

$$\tilde{R} = e^{-i\sqrt{\omega^2 - m_A^2}r^*} + \mathcal{R} e^{i\sqrt{\omega^2 - m_A^2}r^*}, \quad r^* \to \infty \tag{4.10}$$

with the $\mathcal{T}(\mathcal{R})$ the transmission (reflection) amplitudes. Here we must assume that ω, for the bound case of $\omega < m_A$, lies on the physical sheet of $0 \le \arg(\omega^2 - m_A^2)^{1/2} < \pi$. Requiring ingoing waves upon a rotating black hole whose angular velocity Ω is given by (2.10), one must impose an ingoing (negative) group velocity $v_{\rm gr}$ for the wave packet. Then, we choose an ingoing mode near the even horizon as

$$[e^{-i\omega t}\tilde{R}]_{\rm in} = \mathcal{T} e^{-i\omega t} e^{-i(\omega - m\Omega)r_*}. \tag{4.11}$$

Since $V(r,\omega)$ is real, the Wrongskian $W(\tilde{R}, \tilde{R}^*)$ of the complex conjugate solutions of $\tilde{R}$ and $\tilde{R}^*$ satisfies [13]

$$i\frac{d}{dr^*}W(\tilde{R}, \tilde{R}^*) = 0 \tag{4.12}$$

which implies that

$$|\mathcal{R}|^2 = 1 + \left[\frac{m\Omega}{\omega} - 1\right]|\mathcal{T}|^2. \tag{4.13}$$

Here, if the frequency ω of the incident wave satisfies the inequality

$$\omega < m\Omega\,, \tag{4.14}$$

one has $|\mathcal{R}| > 1$, implying that the reflected (scattered) wave is being amplified [27, 15]. Thus, in this superradiance regime, waves appear as outgoing to an observer at infinity, and energy radiated away to infinity actually exceed the energy present in the initial perturbation. Feeding back the amplified scattered waves, one may gradually extract the rotational energy of the $f(R)$-rotating black hole. Press and Teukolsky have suggested to use this mechanism to the black-hole bomb [14]. If one surrounds the black hole by a reflecting mirror, the wave will bounce back and forth between black hole and mirror, amplifying itself each time. To this end, nature may provide its own mirror [28, 30].

It is well known that if one considers a massive scalar field with mass μ scattered off a rotating black hole, then for $\omega < \mu$, the superradiance has unstable modes because the mass term effectively works as a reflecting mirror [31, 18, 32, 33, 34, 35]. As was shown in Fig. 15 of Ref.[13], a shape (ergoregion-$\sim$-mirror) of potential $V(r,\omega)$ has the local maximum as well as the local minimum far from the black hole which generates a secondary reflection of the wave the reflected from the potential barrier. This implies that the meta-stable bound states of $Mm_A \sim r_+/\lambda_c$ with λ_c the Compton wavelength of the massive particle could be formed in the valley of the local minimum. The secondary reflected wave will be reflected again at the far region. Since each scattering off the barrier in the superradiant region increases the amplitude of the wave, the process of reflections will continue with the increased energies of waves, leading to an instability. In our case, the mass m_A of scalaron works as a reflecting mirror. Considering a massive scalaron with mass m_A scattered off $f(R)$-rotating black hole, then for $\omega < m_A$, the mass term effectively works as a reflecting mirror. Similarly, the Kerr black hole was shown to be unstable when choosing $f(R) = R + hR^2$ [36] which is included as a limited form (2.6) when setting all $a_n = 0$ for $n \geq 3$, $a_1 = 1$, and $a_2 = h$.

The instability time scale τ associated with the dynamics of a massive scalar was restricted to two limiting cases: $Mm_A \ll 1$ provides $\tau \simeq 24(a/M)^{-1}(Mm_A)^{-9}(GM/c^3)$ [18], while $Mm_A \gg 1$ indicates $\tau \sim 10^7 e^{1.84Mm_A}(GM/c^3)$ [32]. Recently, the authors [35] have

shown that for $Mm_A \leq 0.42$ and $a = 0.99$, the maximal instability growth rate is given by $\tau^{-1} \simeq 1.5 \times 10^{-7}(GM/c^3)^{-1}$.

5 Discussions

It was suggested that perturbed Kerr black holes discriminates Einstein and $f(R)$ gravities [12]. To this end, we have made a significant progress if the scalaron approach (scalar-tensor theory) represents truly the $f(R)$ gravity including the fourth order differential equation.

It turned out that the $f(R)$ black hole is stable against external perturbations if the scalaron does not have a tachyonic mass ($m_A^2 = f'_A(0)/3f''_A(0) > 0,\ f''_A(0) > 0$) [25]. This is consistent with other perturbation analysis: the Dolgov-Kawasaki instability with $f''(R) < 0$ in cosmological perturbations [37]. For the $f(R)$-AdS black hole, its stability is guaranteed if the scalaron mass squared $\tilde{m}_A^2$ satisfies the Breitenlohner-Freedman bound ($\tilde{m}_A^2 \geq m_{\rm BF}^2 = -9/4\ell^2$) [26]. However, for $f(R)$-rotating black hole, it is unstable even for the positive scalaron mass squared $m_A^2 > 0$ because of its superradiant instability (black hole bomb). We would like to stress that this is a meaningful result for the rotating black hole obtained from a limited form (2.6) of $f(R)$ gravity.

Given the results for the e-folding time $\tau = 10^7 e^{1.84 Mm_A}(GM/c^3)$ for $Mm_A \gg 1$, when might the instability be significant [18, 35]? One assumes that if the scalaron is a pion around a solar-mass black hole, $Mm_A \sim 10^{18}$ with $M \simeq 10^{30}$kg. Then, one concludes that the instability growth rate τ^{-1} is not significant for astrophysical black holes, unless there exists an unknown particles with a tiny but nonzero rest mass. That is, the instability growth rate τ^{-1} is always small for the standard model particles, in compared to the decay rate of particle or the Hawking evaporation rate of the black hole. However, the instability may be important for a pion around a small primordial black hole ($M \leq 10^{12}$kg).

Finally, $f(R)$ theories are supposed to be viable only when the matter is present outside the star/black hole so that the Chameleon mechanism could take place. However, in order to set matter (even a small amount of matter) outside the star/black hole, it will change the results of analysis for the $f(R)$ theories, as the scalaron δA becomes massive.

In conclusion, we have shown that the $f(R)$-rotating black hole arisen from a limited form (2.6) of $f(R)$ gravity is unstable because of the superradiant instability known as the

black-hole bomb idea.

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